

ANALYSIS OF DESIGN EFFECTS AND VARIANCE COMPONENTS IN MULTI-STAGE SAMPLE SURVEYS

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1. INTRODUCTION

a) General

Survey sample techniques have been in use for many years, and among probability samples, there are many types. They vary from simple random sampling to a complex multi-stage stratified sample design and it is the latter type which is the subject of this paper.

b) Need for Multi-Stage Sample Design

A particular survey design is determined by a variety of factors such as objectives of the survey, methods of enumeration, availability of census data, reliability of estimates required, and frequency of survey. The availability of funds usually plays an important, practical part in determining a survey design.

Any sample design, whether it uses personal interviews solely, or in conjunction with telephone or mail, requires clustering of the sample to some degree. Even mail surveys with personal follow-up of non-respondents may require clustering to a certain extent. Although clustering usually reduces the survey cost, the sampling variance of estimates of most statistics is increased. In order to introduce clustering in a survey design, a certain amount of information about the population is required. Census data may be used, for example, to delineate strata and primary sampling units, while more up-to-date information on areas within primary sampling units may be used to delineate sub-sampling units in subsequent stages.

The periodicity or frequency of a particular survey affects the type of survey design and the degree of clustering. For example, a continuous survey usually requires a permanent organization and permanent interviewers who must be trained and controlled.

c) Need for Components of Variance

For the sole purpose of obtaining measures of reliability of estimates, usually confined to estimates of sampling variability, it is self-necessary to split up the variance into components and to estimate each component.

In a multi-stage design however, when total variances are studied, questions frequently arise as to the contribution of various stages to the total variance. Examination of the relative magnitude of variance components throws some light on the alternative sample design through changes in the sizes of the units, or changes in the allocation of sample units that might be adopted to decrease the sampling variance for a given budget, or decrease the cost for a given sampling variance. Clearly, the answer to either of these requires a cost analysis in conjunction with a variance analysis and we are aware that by providing variance analysis without cost analysis the study is incomplete. However, the magnitude of the components at certain

stages may determine the desirability or necessity of isolating cost components at the same stages. In some instances, a variance component may be unacceptably high regardless of the corresponding cost component.

Total sampling variances and design effects [7] enable us to evaluate the gains or losses as a result of stratification, clustering and ratio estimation compared with simple random sampling and simple estimation. However, in order to analyze the design effect more fully, components of variances are required. For example, low design effects may mean that clustering plays no part in increasing the sample variance so that components of variance other than those between ultimate units may be quite low, while high design effects may indicate an extensive clustering effect which would lean toward high components of variance in at least one other stage besides the between ultimate unit component. Also, the size of the sampling unit at a given stage of sampling usually affects the variance component, since the population variance of which the variance component is a function, almost always increases with the size of a unit faster than by a linear relationship with the size. The reason for the faster than linear increase is a positive intra-class correlation [5]. Design effects by stages of sampling may also be calculated (See Definition in Section 4b).

2. VARIANCE COMPONENTS AND ANALYSIS

Variance components were undertaken for about 40 characteristics in June, 1973, for the Canadian Labour Force Survey, using Yates-Grundy estimation formulas [9], and further developed by Gray [3] and [4]. More details on the sample design of the Canadian Labour Force Survey may be obtained from [2] and [8], and the details related to variance component estimation may be obtained from [4].

The study of variance components based on the results from the LFS lead to two basic types of analysis, (a) a study of the variance components as percentages of the total variance and their relationship to design effects [7], and (b) a study of a variance function in terms of average weights and numbers of units at various stages and the population variances for which individual design effects may be obtained.

3. DESIGN EFFECTS AND CATEGORY OF CHARACTERISTICS

On the above broad definition of the types of characteristics as well as the observed design effects, the 19 characteristics were grouped into three categories, maintaining as much as possible the common category between urban and rural areas.

There is a rather high correlation of .860 between the design effects of the characteristics for urban and rural areas and without either "Finance and Insurance" or "Non-Agriculture Employed", the correlation increases to .907. The design effects in the urban and rural areas have also been ranked in order to show more clearly the correlation between the two sets of observations. In general, the clustering effect as seen by the magnitude of the design effect is more significant in the rural areas than in the urban areas, the average design effects being 1.604 and 2.020 in the urban and rural areas respectively. Although observations pertaining to category I are lacking in this study, monthly variance estimates of other categories using the Keyfitz formula [6] have revealed a similar phenomenon of low design effects among characteristics pertaining to specific age-sex categories.

4. ANALYSIS OF COMPONENTS OF VARIANCE

a) General

Each stage of sampling results in some contribution to the total sampling variance. The magnitude of the components vary from stage to stage and by characteristic within each stage of sampling. These have been calculated using methods developed in [4] for many Labour Force Survey characteristics in small urban and rural areas across Canada where four stage sampling was applied. The results are given in Table 2 for Urban and Rural. The percentages of the components of variance are studied in relation to the average weights and sizes of sampling units at the various stages of selection.

(b) Observation from Table 2 Dealing With Variance Components

- i) The average weight \bar{W}_r (i.e. the total number of rth stage units within all sampled (r-1)th stage units divided by the sampled number of units within all sampled (r-1)th stage units, "0" being taken as the stratum level) for each stage of sampling.
- ii) The average size of rth stage unit \bar{P}_r and averaged over the characteristics of each category.
- iii) The average percentage contribution for each component of variance $100 V_r/V$ by stage of sampling.
- iv) The average design effect F_r , defined by
$$V_r / \left(\sum_{s=1}^r \bar{W}_s - 1 \right) Ppq$$
 and with some further approximation may be shown to equal $\bar{\sigma}_{r:r-1}^2 / \bar{Ppq}$.
- v) The average measure of homogeneity, $\bar{\delta}_{4:r}$, referring to the measure of similarity in characteristics for any pair of households within rth stage units of a stratum as compared with any pair of households within the stratum, averaged over all strata across Canada.

The items above are tabulated for Canada urban and Canada rural as of June, 1973.

$\bar{\sigma}_{r:r-1}^2$ is the average population variance between rth stage units within (r-1)th stage units (not shown in these tables) averaged over all (r-1)th stage units. All averages mentioned above are estimates based on the sample.

In a particular (r-1)th stage unit i_{r-1} ,
$$N_{r|i_{r-1}} \sigma_{r|i_{r-1}}^2 = \sum_{i_r} p_{i_r|i_{r-1}} (x_{i_r|i_{r-1}} / p_{i_r|i_{r-1}} - x_{i_{r-1}})^2$$
 where $N_{r|i_{r-1}}$ is the

number of rth stage units in i_{r-1} , $p_{i_r|i_{r-1}}$ is

the relative size of rth stage unit i_r , and

$x_{i_r|i_{r-1}}$ is the characteristic value of unit i_r

in i_{r-1} such that $\sum_{i_r} x_{i_r|i_{r-1}} = x_{i_{r-1}}$.

In the above, $i_0 = h$ (stratum); $i_1 = (\text{PSU})$,

$i_2 = K$ (segment in type of area j); $i_3 = c$

(cluster) and $i_4 = \text{household}$.

c) Analysis of Table 2

The percentage contribution to the total variance, design effect by stages, and measures of homogeneity all show a distinct pattern by the three categories of characteristics in both urban and rural areas.

Turning first to the percentage contribution, we find that the total variance for category I characteristics is almost entirely contributed by the between household component (80% in the urban area, and 59% in the rural area). For category II characteristics, the between household component of variance drops to 68% in the urban area and 47% in the rural area with slight changes to moderate increases in the other components. For category III characteristics, the between household component drops to 44% and 30% in the urban and rural areas respectively, but the between PSU component increases to about 1/3 in both types of areas.

The high between household components of variance is reflected in low design effects and low measures of homogeneity at all stages. However, looking at individual characteristics for the moment rather than the averages, one notices that high components of variance at a given stage are reflected in high design effects and high measures of homogeneity. Negative components of variance estimate and negative design effects may be interpreted as estimates of unknown values that are positive but close to zero.

If measures of homogeneity are calculated, design effects may be considered redundant. However, measures of homogeneity are difficult to calculate and apply in the formulae for variance functions and population variances must be calculated individually (not averaged over characteristics), since they are a function of the size of

the characteristic total or mean. Design effects are useful both for individual characteristics and for sets of characteristics to arrive at estimated population variances $\sigma^2_{r:r-1}$ in the components of variance functions.

5. VARIANCE COMPONENTS AND DESIGN EFFECTS IN ALLOCATION STUDIES

For a given design (i.e. strata, units with their sizes unchanged), the population variances remain unchanged and the variance function (#7 Appendix) will be a function of the average weights for sampling at the various stages only.

Or, algebraically, the r th stage component of variance is given by:

$$V_r = \left[\prod_{s=1}^{r-1} \bar{w}_s (\bar{w}_r - 1) / \prod_{s=1}^{r-1} \bar{w}_s (\bar{w}_s - 1) \right] V_r''$$

where $''$ denotes the present value of the parameters. For example,

$1/\bar{w}_1''$ = present sampling fraction of PSUs and

V_1'' = present between PSU component of variance.

For a 4-stage sample design, $V = \sum_{r=1}^4 V_r$, and by

substituting the current percentage variance components for different weights, one can readily see whether the altered weights would increase or decrease the variance according as V is greater or less than 100. Table 3 provides the variances that would occur under different sample allocations of weight changes while maintaining the same overall weight (product of the 4 weights kept the same). Fifteen different values of $\sum_{r=1}^4 V_r / \sum_{r=1}^4 V_r''$ are obtained for

each of the three categories using different component weights such that $\prod_{s=1}^4 \bar{w}_s = \prod_{s=1}^4 \bar{w}_s''$.

The weights are varied mostly by halving and doubling the number of selected units per unit of the next lower stage for two of four stages (indicated by .5* and .2*). One special case of a census of primary sampling units with a more scattered sample within is also considered. In Table 3, the variance as compared with the present are obtained for Canada Urban and Canada Rural.

The total cost of the survey will not be the same for all of the allocations presented in Table 6, since the households may be on an average more spread out or more clustered than at present, but it serves to illustrate one method of optimization on the assumption that the total survey costs may not differ appreciably if the total sample size is kept constant, whatever the allocation by stages.

Under the survey constraints of a fixed overall sample size and fixed strata and delineated sampling units, it turns out that in only one of the 15 cases examined in Table 3 will the total variance be reduced for all 3 categories of

characteristics, although striking reductions may occur in any one of the categories. If every PSU is taken (i.e. each PSU a stratum), then the variance of category III variables would be substantially reduced, but 1/2 of the segments and 1/2 of the clusters and the appropriate proportion of households within clusters to maintain the same overall sampling ratios (see note at bottom of Table 3) would be required. There is a relatively small choice of allocations to examine. \bar{w}_r cannot exceed \bar{P}_{r-1}/\bar{P}_r , for we would take fewer than one r th stage unit per $(r-1)$ th stage unit. Also \bar{w}_r cannot be less than one, for we would be taking more units in the sample than there exist in the population. It would be possible to take twice as many rural segments per PSU as we now do, but not twice as many urban segments, since in most PSUs we already take every urban segment. For this reason, the cases of 2* for segments were omitted from Table 3.

The change in the variance from the present under different allocations given in Table 3 (no changes in the sizes or delineations of the units at the various stages) is given by:

$$V - V'' = \sum_{r=1}^4 \left[\prod_{s=1}^{r-1} \frac{a_s^{-1}}{\bar{w}_s} (\bar{w}_r^{-1} \bar{w}_r'' - 1) / \prod_{s=1}^{r-1} \bar{w}_s'' (\bar{w}_s'' - 1) - 1 \right] V_r''$$

where \bar{w}_r'' denotes the present weight (inverse sampling ratio) for sampling r th, stage units, $a_r^{-1} \bar{w}_r''$ denotes the altered weight according to the allocation strategy as of Table 3 (commonly $a_r = .5$ or $.2$), and V_r'' denotes the present variance between r th stage units, expressed as a present total variance. By substituting the appropriate values of a_r^{-1} in the above formula, simple relationships between V and V'' may be derived for each allocation strategy and the conditions between V_r'' 's necessary to ensure a decrease in the variance readily derived. The simplified relationship may help explain peculiar results such as allocation #4 and #5, where it turns out that $V_4 < 4.1 V_3$ in order to ensure a decrease in V from V'' in the urban area.

Conclusions from Table 3

- i) Despite the existence of take-all clusters, the between household component of variance accounts for a significant portion of the total variance and in fact dominates the total variance for category I variables (e.g., 80% of the total variance in Canada urban). Consequently, for fixed total sample size, changes in the allocation will have a small effect on the variance for category I variables.
- ii) If we select only one PSU instead of 2 as in allocations #6 and 7, the variances of category II and III variables would increase substantially, in some cases over

50%. While the variance is substantially increased, the field costs might be only marginally reduced since the workload in each PSU is doubled and we might require two interviewers as we would in the case of 2 selected PSUs. If, however, the interviewer's workload could be doubled without jeopardizing the quality of the interviews, then there would be some reductions in the cost with fewer interviewers to train and lower travel costs.

- iii) If we select 3 or 4 PSUs as in the case of allocations 8 to 12 and 13 to 15 respectively, there are only small decreases or increases for Category II Urban but a greater tendency for reductions for Category II Rural. For Category III Urban, the tendency leans towards reduced variance up to 20% (exceptions being allocation 11 and 12) but for Category III Rural, the reductions vary from a negligible 3% in the case of allocation 11 and 12 to about 30%. The interview costs would likely increase substantially either through increased travel between more PSUs which would result in larger areas to cover or through an increase in the number of interviewers in smaller areas and smaller workloads for each, thus increasing the training and interviewer control costs.
- iv) If, in the extreme case, we select every PSU in each stratum, the reductions in the variance for Category II Urban and Category III variables are large (e.g., 45% reduction for Category III in the rural area). The effect on Category I variables and Category II in the urban areas is minimal (less than 8%). If we had delineated strata the size of PSUs, there would perhaps have been a larger reduction than indicated in this paper. For the philosophy behind the delineated strata is to make them as distinct as possible while PSUs should be as much alike as possible.
- v) Despite the rather severe constraint of a fixed overall sample size with varying weights by stages but with delineated strata and units unchanged, many interesting and important results were revealed. The data of Tables 2 and 4, and the variance function in terms of weights could be further utilized to determine the variance for the requirements of reducing or increasing the total sample size, examining various strategies of reduction or increase of the number of selected units at the different stages of selection.

6. VARIANCES OF CHARACTERISTICS AND SIZES OF UNITS

Just as in Section 5, we examined the changes in the variances as the allocation of the sample by stages (maintaining or fixed overall sample size and the same delineated units and strata), we can examine the changes in the variances as the average sizes of the units are altered (maintaining the same strata and sampling rates by

stages). In a similar manner employed in Section 5, the average sizes of the units are halved or doubled in such a manner that a lower stage unit would never be smaller than a higher stage unit.

Using #2 in the methodology, V was obtained for different average sizes of \bar{P}_r 's for $r = 1, 2$, and 3 while \bar{P}_0 and \bar{P}_4 are fixed. The results are presented in Table 4.

Analysis of Table 4

It can be seen that only marginal improvements are observed for any strategy of altered sizes of units and they are largely confined to the case of changed segment and cluster sizes. When the PSU size is doubled, however, the rural estimates are shown to possess an increase in variance up to almost 20% for Category I variables and almost 40% for Category III variables. In the case of Category III variables with high measures of homogeneity for most sizes of units, any increase in the average size of units will tend to increase rather than decrease the variance even while maintaining the fixed overall sample size. From the observed large between PSU variance of rural Category III variables, one would expect some substantial reductions in the variance of these variables if smaller PSUs were delineated and twice as many of them were selected. The interview costs for smaller PSUs, however, may be greater on a per household basis because of the extra spread of the sample or the necessity to hire extra interviewers to avoid the travelling between PSUs.

General Conclusion

The analysis of the components of variance pertained strictly to the current LFS design. However, the methods can be easily adopted to any multi-stage sample design similar to the above, such as the revised LFS design. Only one survey's data was used in this article and it appears that the allocation of the sample by stages and the size of delineated units are near optimum values for the given sample size. However, the cost function for changes in the weights and/or sizes of units was not taken into account so that one cannot draw fully certain conclusions about the optimum properties of the sample design. To monitor the sample design for its efficiency in terms of allocation and size of units continuously in a continuing survey where growth and features are constantly changing it is recommended that components of variance be obtained four times per year across Canada with special runs for smaller areas where problems are occurring.

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Table 1: Average Design Effects by Category (no. of characteristics in brackets*) for Canada Urban and Canada Rural

Category	Canada Urban	Canada Rural
1	.864 (2)	.989 (2)
2	1.141 (9)	1.529 (10)
3	2.310 (8)	3.018 (7)

* with only one exception, the category of the characteristic was the same in urban and rural areas

Table 2: Average weight (wt.), size of unit; percent component of variance %, design effect F, and measure of homogeneity δ by stage of sampling for Canada by type of area and by Category of variable (June, 1973).

Type of Area, Stage of Sampling	Aver. Wt.	Aver. Size	Category I			Category II			Category III		
			%	F	δ	%	F	δ	%	F	δ
Urban, 0	1.00	16,053									
1	6.52	1,232	6.08	1.385	.0013	10.49	3.736	.0044	33.57	21.71	.0249
2	1.24	492	1.87	0.958	.0044	3.81	2.818	.0129	8.80	13.70	.0646
3	4.36	34.9	11.12	0.562	-.0235	17.58	1.113	.0169	13.46	1.718	.0807
4	4.16	2.33	80.95	1.002	1.0000	68.07	1.061	1.0000	44.18	1.339	1.0000
Rural, 0	1.00	24,180									
1	5.98	2,020	11.10	3.035	.0037	14.44	6.801	.0056	37.17	29.75	.0214
2	2.77	332	5.63	0.673	.0036	20.79	4.084	.0242	19.17	7.616	.0488
3	3.53	17.9	24.49	0.859	-.0091	18.20	0.946	.0055	13.93	1.473	.0530
4	2.47	2.27	58.78	0.930	1.0000	46.57	1.036	1.0000	29.73	1.273	1.0000

Methodology For The Study

As mentioned earlier, the variance components were estimated by the Yates-Grundy formulas [9], also quoted in Fellegi [1] and the approximate variance function in terms of weights and population variances, developed in more detail by Gray [3] and [4] is given by:

$$V_r = (P/\bar{P}_r) \cdot \bar{W}_1 \bar{W}_2 \dots \bar{W}_{r-1} (\bar{W}_r - 1) \sigma_{r:r-1}^2 / (1 - \bar{P}_{r-1}/\bar{P}_r) \quad (1)$$

= rth stage component of variance, where

P = total population

\bar{P}_r = average size of rth stage unit

\bar{W}_s = average weight for sth stage units within (s-1)th stage units

$\sigma_{r:r-1}^2$ was defined in the text.

If the strata and sampling units at all stages remain fixed as in Table 3, then

$$V_r \propto \prod_{s=1}^{r-1} \bar{W}_s (\bar{W}_r - 1). \text{ Although the variance estimates were obtained using the appropriate individual and joint selection probabilities, the variance function was developed on the assumption that the average f.p.c. } r_{FP} = -1/(\bar{N}_{r:r-1} - 1),$$

where $\bar{N}_{r:r-1} = \bar{P}_{r-1}/\bar{P}_r$ is the average number of rth stage units per (r-1)th stage unit.

To study the changes in the variances as the sizes of the units are changed, but maintaining fixed household and strata sizes, as in Table 4, the population variance function was derived below.

$$\sigma_{r:r-1}^2 = \bar{N}_{4:r} \sigma_{4:o}^2 \{ [1 + (\bar{N}_{4:r-1} - 1) \bar{\delta}_{4:r-1}] - \bar{N}_{r:r-1}^{-1} [1 + (\bar{N}_{4:r-1} - 1) \bar{\delta}_{4:r-1}] \} \quad (2)$$

where $\bar{N}_{s:r}$ = average no. of sth stage units per rth stage unit.

Estimates of $\bar{\delta}_{4:r}$ were obtained from June, 1973 data and interpolated by the reciprocal of the average size of units as different sizes of units at various stages were considered.

Table 3: Variance/Present Variance using different allocation of weights while maintaining current overall weight

Alloc. No.	Sample Allocation				Category I		Category II		Category III	
	PSUs/ Str	Seg/ PSU	Clus/ Seg	Hhld/ Clus	Urban	Rural	Urban	Rural	Urban	Rural
1)	*(2)	*	*	*	100.00	100.00	100.00	100.00	100.00	100.00
2)	*	.5*	2*	*	106.35	99.13	114.45	125.34	141.46	124.49
3)	*	.5*	*	2*	95.17	93.31	115.71	119.06	114.95	123.71
4)	*	*	.5*	2*	88.82	94.18	101.26	93.71	103.49	99.21
5)	*	*	2*	.5*	105.59	102.91	99.37	103.41	98.26	100.39
6)	.5*(1)	*	2*	*	105.74	109.28	110.97	130.94	144.45	158.30
7)	.5*	*	*	2*	94.56	103.46	112.22	124.65	147.93	157.51
8)	1.5*(3)	2/3*	*	*	100.20	96.62	101.17	98.13	99.04	88.71
9)	1.5*	*	2/3*	*	98.08	96.33	96.35	106.57	85.21	96.87
10)	1.5*	*	*	2/3*	101.81	98.84	95.93	91.78	84.05	80.81
11)	1.5*	.5*	4/3*	*	102.32	96.33	105.99	106.57	112.87	96.87
12)	1.5*	.5*	*	4/3*	98.59	94.39	106.41	104.48	114.04	96.61
13)	2.*(4)	.5*	*	*	100.31	94.93	101.74	97.20	98.51	83.10
14)	2.*	*	.5*	*	97.13	95.36	94.52	84.53	77.78	70.85
15)	2.*	*	*	.5*	102.72	98.27	93.89	87.68	76.03	71.25
16)	all	.5*	.5*	(a)	100.43	94.05	92.44	80.56	67.38	55.81

Number of PSUs selected per stratum in brackets

(a): $\bar{w}_4 = 6.78(U); 3.69(R)$

*: Means current sampling ratio of selected units within units at the next lower stage on an average.
all: Means that there is no sampling of units at a particular stage, i.e. all units are selected.

Table 4: Variance/Present Variance for different size of units by stages of sampling (strata unchanged) and the same weights applied at each stage as currently

Strategy Number	Sizes of Units Compared to Current			Category I		Category II		Category III	
	PSU	Seg	Clus	Urban	Rural	Urban	Rural	Urban	Rural
1	1.*(2)	1.*	1.*	100.00	100.00	100.00	100.00	100.00	100.00
2	1.*	.5*	1.*	99.258	100.242	98.190	91.136	94.980	91.472
3	1.*	1.*	.5*	108.168	105.923	102.586	105.329	100.622	102.212
4	1.*	2.*	1.*	100.793	99.936	101.777	101.790	105.131	101.733
5	1.*	1.*	2.*	98.526	99.177	100.106	98.913	100.465	99.889
6	1.*	.5*	.5*	107.937	106.560	101.109	96.789	95.747	93.865
7	1.*	.5*	2.*	97.784	99.548	98.356	90.221	95.444	91.360
8	1.*	2.*	.5*	108.307	105.693	103.318	106.073	103.083	103.003
9	1.*	2.*	2.*	100.193	99.064	103.803	102.746	111.092	103.396
10	2.*(1)	.5*	1.*	98.568	112.633	104.214	99.607	98.400	127.782
11	2.*	1.*	.5*	107.393	118.124	108.082	113.193	103.488	137.958
12	2.*	2.*	1.*	100.017	111.678	106.289	109.131	107.997	137.029
13	2.*	1.*	2.*	97.750	111.376	105.617	106.971	103.327	135.636
14	2.*	.5*	.5*	102.809	118.950	107.132	105.259	99.173	130.170
15	2.*	.5*	2.*	97.094	111.809	104.378	98.712	98.861	127.750
16	2.*	2.*	.5*	107.531	117.436	107.873	113.415	105.949	138.300
17	2.*	2.*	2.*	99.417	110.806	108.372	110.088	113.849	138.693

A * means A times as large as current average size.

No. of selected PSUs per stratum in brackets